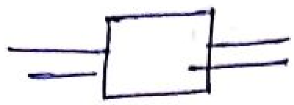
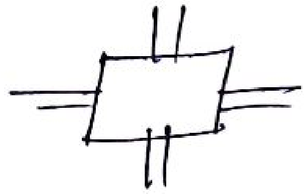
# 4. Two-port Networks

port is having one receiving end and one sending end.

two port means two sending ends and two receiving ends.



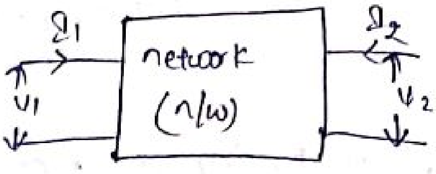
suppose we taken n port network has 'n' no. of output ports and 'n' no. of input port.



two port network parameters are classified into

1. Z- parameters / open circuit parameters.
2. Y- parameters / short circuit parameters
3. ABCD- parameters / transmission parameters
4. H- parameters
5. inverse ABCD- parameters
6. inverse H- parameters / g- parameters.

i) Z - parameters:-




$(V_1, V_2) = f(I_1, I_2)$

$V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow (1)$

$V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow (2)$

If $I_2 = 0$ i.e., port-2 is open circuit

from eqn(1) & (2)

$V_1 = Z_{11} I_1 + Z_{12}(0)$

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

and $V_2 = Z_{21} I_1 + Z_{22}(0)$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$$

If $I_1 = 0$ i.e., port-1 acts as open circuit

from eqn(1) & (2)

$V_1 = Z_{11}(0) + Z_{12}(I_2)$

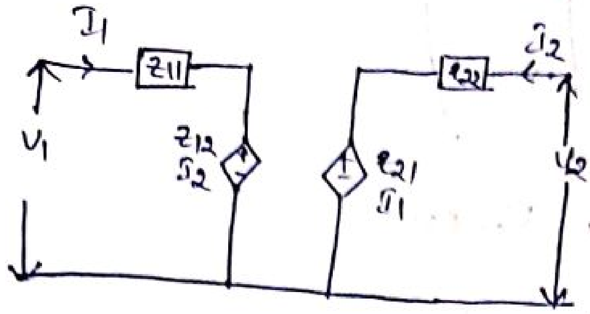
$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

and $V_2 = Z_{21}(0) + Z_{22}(I_2)$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

## Equivalent circuit :-

from equation (1) & (2)




$Z_{11}$ = driving point impedence at port-(1)

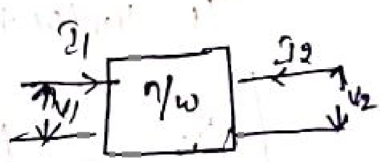
$Z_{12}$ = forward impedence

$Z_{21}$ = reverse impedence

$Z_{22}$ = driving point impedence at port-2

## Y- parameters :-

$(I_1, I_2) = f(V_1, V_2)$



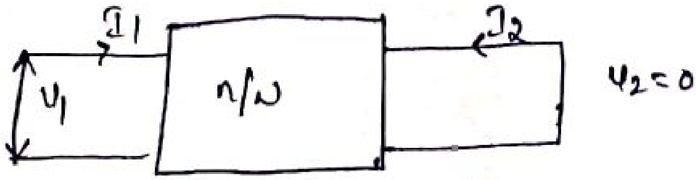

$I_1 = Y_{11} V_1 + Y_{12} V_2 \longrightarrow (1)$

$I_2 = Y_{21} V_1 + Y_{22} V_2 \longrightarrow (2)$

in matrix-form

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

if $V_2 = 0$ i.e, port-2 is short circuit



from eqn(1) & (2)

$$I_1 = Y_{11} V_1 + Y_{12}(0)$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

and $I_2 = Y_{21} V_1 + Y_{22}(0)$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

if $V_1 = 0$ ; the port-1 is short circuit.


from eqn(1) & (2)

$$I_1 = Y_{11}(0) + Y_{12} V_2$$

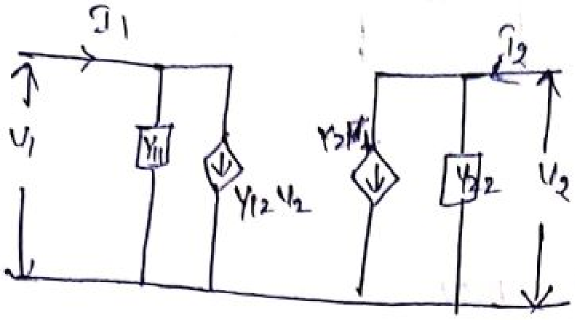
$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

and $I_2 = Y_{21}(0) + Y_{22} V_2$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

Equivalent circuit:

from eqn (1) & (2)




$Y_{11}$ = driving point admittance at port-1

$Y_{12}$ = forward admittance

$Y_{21}$ = Reverse admittance

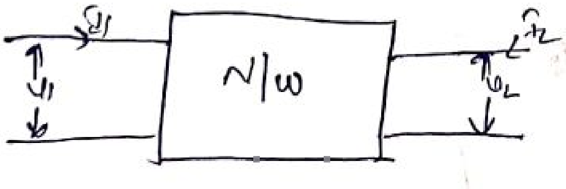
$Y_{22}$ = driving point admittance at port-2

ABCD-parameters:-

$(V_1, I_1) = f(V_2, -I_2)$

$V_1 = AV_2 - BI_2 \rightarrow (1)$

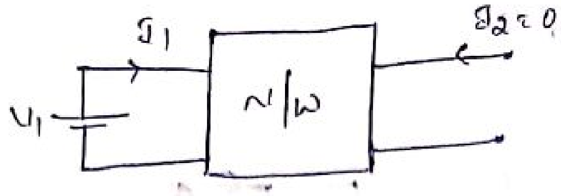
$I_1 = CV_2 - I_2 D \rightarrow (2)$




in matrix form

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

If $I_2 = 0$; i.e., port-2 as open circuit




from eqn(1) & (2)

$$V_1 = AV_2 - B(0)$$

$$A = \frac{V_1}{V_2}\Big|_{I_2=0}$$

and $I_1 = CV_2 - B(0)$

$$C = \frac{I_1}{V_2}\Big|_{I_2=0}$$

If $V_2 = 0$; i.e., port-2 as short-circuited.


from eqn(1) & (2)

$$V_1 = A(0) - BI_2$$

$$B = \frac{-V_1}{I_2}\Big|_{V_2=0}$$

and $I_1 = C(0) - D(I_2)$

$$D = \frac{-I_1}{I_2}\Big|_{V_2=0}$$

Equivalent circuit:

ABCD parameters:-

A ⇒ reverse voltage gain

D ⇒ reverse current gain

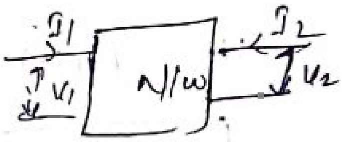
B ⇒ forward impedence

C ⇒ forward admittance

h-parameters:-

$(V_1, I_2) = f(I_1, V_2)$

$V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (1)$

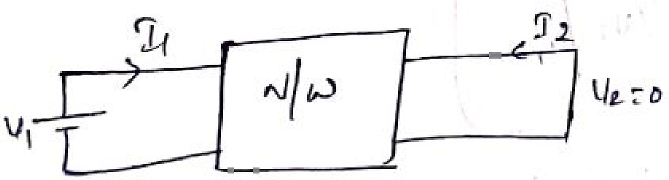



$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (2)$

In matrix form

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

If $V_2 = 0$; i.e., the port-2 was short circuit




from eqn (1) & (2)

$V_1 = h_{11} I_1 + h_{12}(0)$

$$h_{11} = \frac{V_1}{I_1}\Big|_{V_2 = 0}$$

and

$$I_2 = h_{21} I_1 + h_{22}(0)$$

$$h_{21} = \frac{I_2}{I_1}\Big|_{V_2 = 0}$$

If $I_1 = 0$; the port-1 acts as open circuit


$$V_1 = h_{11}(0) + h_{12}(V_2)$$

$$h_{12} = \frac{V_1}{V_2}\Big|_{I_1 = 0}$$

and

$$I_2 = h_{21}(0) + h_{22} V_2$$

$$h_{22} = I_2/V_2 \Big|_{I_1 = 0}$$

$h_{11}$ ⇒ forward impedence

$h_{12}$ ⇒ forward voltage gain

$h_{21}$ ⇒ Reverse current gain.

$h_{22}$ ⇒ backward impedence.

## Relations:-

* Z-parameters in terms of Y-parameters :-

$$Z = \frac{1}{Y} = [Y]^{-1}$$

$$= \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

$$= \frac{1}{Y_{11}Y_{22} - Y_{12}Y_{21}} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}, \quad \Delta Y = \frac{1}{Y_{11}Y_{22} - Y_{12}Y_{21}}$$

$$Z_{11} = \frac{Y_{22}}{\Delta Y}$$

$$Z_{12} = \frac{-Y_{12}}{\Delta Y}$$

$$Z_{21} = \frac{-Y_{21}}{\Delta Y}$$

$$Z_{22} = \frac{Y_{11}}{\Delta Y}$$

* Z-parameters in terms of ABCD-Parameters

we know that Z-parameters equations

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow (2)$$

and ABCD-parameters,

$$V_1 = A V_2 - B I_2 \longrightarrow (3)$$

$$I_1 = C V_2 - D I_2 \longrightarrow (4)$$

$(4) \Rightarrow$ $$C V_2 = I_1 + D I_2$$

$$V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2 \longrightarrow (5)$$

substitute (5) in (3)

$$V_1 = A\left[\frac{1}{C} I_1 + \frac{D}{C} I_2\right] - B I_2$$

$$= \frac{A}{C} I_1 - \left(\frac{BC - AD}{C}\right) I_2 \longrightarrow (6)$$

Comparing (1) & (6) Equations

$$Z_{11} = A/C \quad , \quad Z_{12} = \left[\frac{-BC + AD}{C}\right]$$

Comparing (2) & (5) Equations

$$Z_{21} = \frac{1}{C} \quad , \quad Z_{22} = +D/C$$

* z-parameters interms of h-parameters:

we know that z-parameters equations

$$V_1 = z_{11} I_1 + z_{12} I_2 \longrightarrow (1)$$

$$V_2 = z_{21} I_1 + z_{22} I_2 \longrightarrow (2)$$

and h-parameters

$$V_1 = h_{11} I_1 + h_{12} V_2 \longrightarrow (3)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \longrightarrow (4)$$

Eqn (4) $\Rightarrow$ $h_{22} V_2 = I_2 - h_{21} I_1$

$$V_2 = \frac{1}{h_{22}} I_2 - \frac{h_{21}}{h_{22}} I_1 \longrightarrow (5)$$

(3) $\Rightarrow$ $V_1 = h_{11} I_1 + h_{12} \left[\frac{1}{h_{22}} I_2 - \frac{h_{21}}{h_{22}} I_1\right]$

$$V_1 = \frac{h_{11} h_{22} - h_{21} h_{12}}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2 \longrightarrow (6)$$

Comparing (1) & (6) equations

$$z_{11} = \frac{h_{11} h_{22} - h_{21} h_{12}}{h_{22}} \; ; \; z_{12} = \frac{h_{12}}{h_{22}}$$

Comparing (2) & (5) equations

$$z_{22} = \frac{1}{h_{22}} \; ; \; z_{21} = \frac{-h_{21}}{h_{22}}$$

* Y-parameters in terms of Z-parameters:-

Y parameters equation's are

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \rightarrow (1)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \rightarrow (2)$$

$$Y = \frac{1}{Z} = [Z]^{-1}$$

$$= \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{+1}{Z_{11}Z_{22} - Z_{12}Z_{21}} \begin{bmatrix} +Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & \frac{-Z_{12}}{\Delta Z} \\ \frac{-Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$$

$$Y_{11} = \frac{Z_{22}}{\Delta Z}$$

$$Y_{12} = \frac{-Z_{12}}{\Delta Z}$$

$$Y_{21} = \frac{-Z_{21}}{\Delta Z}$$

$$Y_{22} = \frac{Z_{11}}{\Delta Z}$$

$Z_{11} = 3\Omega$ $\Delta Z = 8$

$Z_{12} = 1\Omega$

$Z_{21} = 1\Omega$

$Z_{22} = 3\Omega$

* Y- parameters in terms of ABCD- parameters.

Y-parameter $I_1 = Y_{11} V_1 + Y_{12} V_2 \rightarrow (1)$

$I_2 = Y_{21} V_1 + Y_{22} V_2 \rightarrow (2)$

ABCD- parameter equations:-

$V_1 = AV_2 - BI_2 \rightarrow (3)$

$I_1 = CV_2 - DI_2 \rightarrow (4)$

$(3) \Rightarrow \quad BI_2 = AV_2 - V_1$

$I_2 = \frac{A}{B} V_2 - \frac{1}{B} V_1 \rightarrow (5)$

Substitute (5) in (6)

$I_1 = CV_2 - D\left[\frac{A}{B} V_2 - \frac{1}{B} V_1\right]$

$I_1 = \left[\frac{BC-AD}{B}\right] V_2 + \frac{D}{B} V_1 \rightarrow (6)$

Compare (1) & (6) equations

$Y_{11} = \frac{D}{B}$ ; $Y_{12} = \frac{BC-AD}{B}$

compare (2) & (5) equations

$Y_{21} = -1/B$ ; $Y_{22} = A/B$

"Y-parameters in terms of h-parameters

Y-Parameters Equation's are

$I_1 = Y_{11} V_1 + Y_{12} V_2 \rightarrow (1)$

$I_2 = Y_{21} V_1 + Y_{22} V_2 \rightarrow (2)$

h-parameters Equation's are

$V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (3)$

$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (4)$

$(3) \Rightarrow h_{11} I_1 = V_1 - h_{12} V_2$

$$I_1 = \frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2 \rightarrow (5)$$

$$(4) \Rightarrow I_2 = h_{21}\left[\frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2\right] + h_{22} V_2$$

$$= \frac{h_{21}}{h_{11}} V_1 + \frac{h_{11} h_{22} - h_{21} h_{12}}{h_{11}} V_2 \rightarrow (6)$$

compare (1) & (5) equations

$Y_{11} = \frac{1}{h_{11}}$ ; $Y_{12} = \frac{-h_{12}}{h_{11}}$

compare (2) & (6) equations

$Y_{21} = \frac{h_{21}}{h_{11}}$ ; $Y_{22} = \frac{h_{11} h_{22} - h_{21} h_{12}}{h_{11}}$

ABCD- parameters in terms of z-parameters

ABCD - parameters Equations are:

$$V_1 = A V_2 - B I_2 \rightarrow (1)$$

$$I_1 = C V_2 - D I_2 \rightarrow (2)$$

z-parameters equations are

$$V_1 = z_{11} I_1 + z_{12} I_2 \rightarrow (3)$$

$$V_2 = z_{21} I_1 + z_{22} I_2 \rightarrow (4)$$

$$(4) \Rightarrow z_{21} I_1 = V_2 - z_{22} I_2$$

$$I_1 = \frac{1}{z_{21}} V_2 - \frac{z_{22}}{z_{21}} I_2 \rightarrow (5)$$

$$(3) \Rightarrow V_1 = z_{11}\left[\frac{1}{z_{21}} V_2 - \frac{z_{22}}{z_{21}} I_2\right] + z_{12} I_2$$

$$V_1 = \frac{z_{11}}{z_{21}} V_2 + \frac{z_{12} z_{21} - z_{11} z_{22}}{z_{21}} I_2 \rightarrow (6)$$

compare (1) & (6) equations

$$A = \frac{z_{11}}{z_{21}} \quad ; \quad B = \frac{z_{11} z_{22} - z_{12} z_{21}}{z_{21}}$$

compare (2) & (5) equations

$$C = \frac{1}{z_{21}} \quad ; \quad D = \frac{z_{22}}{z_{21}}$$

* ABCD parameters in terms of Y-parameters:-

ABCD parameters Equations:

$V_1 = AV_2 - BI_2 \rightarrow (1)$

$I_1 = CV_2 - DI_2 \rightarrow (2)$

Y-parameters Equations:-

$I_1 = Y_{11}V_1 + Y_{12}V_2 \rightarrow (3)$

$I_2 = Y_{21}V_1 + Y_{22}V_2 \rightarrow (4)$

(4) $\Rightarrow Y_{21}V_1 = I_2 - Y_{22}V_2$

$$V_1 = \frac{1}{Y_{21}} I_2 - \frac{Y_{22}}{Y_{21}} V_2 \rightarrow (5)$$

(3) $\Rightarrow I_1 = Y_{11}\left[\frac{1}{Y_{21}} I_2 - \frac{Y_{22}}{Y_{21}} V_2\right] + Y_{12}V_2$

$$I_1 = \frac{Y_{11}}{Y_{21}} I_2 + \frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}} V_2 \rightarrow (6)$$

compare (1) & (5) equations

$A = \frac{1}{Y_{21}}$ ; $B = \frac{Y_{22}}{Y_{21}}$

compare (2) & (6) equations

$C = \frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}}$ ; $D = \frac{-Y_{11}}{Y_{21}}$

## ABCD parameters in terms of h-parameters:-

ABCD parameters Equations:-

$$V_1 = AV_2 - BI_2 \rightarrow (1)$$

$$I_1 = CV_2 - DI_2 \rightarrow (2)$$

h-parameters Equations

$$V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (3)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (4)$$

$$(4) \Rightarrow h_{21} I_1 = I_2 - h_{22} V_2$$

$$I_1 = \frac{1}{h_{21}} I_2 - \frac{h_{22}}{h_{21}} V_2 \rightarrow (5)$$

$$(5) \Rightarrow V_1 = h_{11}\left[\frac{1}{h_{21}} I_2 - \frac{h_{22}}{h_{21}} V_2\right] - h_{12} V_2$$

$$V_1 = \frac{h_{11}}{h_{21}} I_2 - \left[\frac{h_{11}h_{22} + h_{12}h_{21}}{h_{21}}\right] V_2 \rightarrow (6)$$

compare (1) & (6) equations

$$A = -\left[\frac{h_{11}h_{22} + h_{12}h_{21}}{h_{21}}\right] ; \quad B = \frac{-h_{11}}{h_{21}}$$

compare (2) & (5) equations

$$C = \frac{-h_{22}}{h_{21}} ; \quad D = \frac{-1}{h_{21}}$$

* h-parameters in terms of z-parameters

h-parameter Equations are:

$$V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (2)$$

z-parameters Equations:

$$V_1 = z_{11} I_1 + z_{12} I_2 \rightarrow (3)$$

$$V_2 = z_{21} I_1 + z_{22} I_2 \rightarrow (4)$$

$$(4) \Rightarrow z_{22} I_2 = V_2 - z_{21} I_1$$

$$I_2 = \frac{1}{z_{22}} V_2 - \frac{z_{21}}{z_{22}} I_1 \rightarrow (5)$$

$$(3) \Rightarrow V_1 = z_{11} I_1 + z_{12} \left[ \frac{1}{z_{22}} V_2 - \frac{z_{21}}{z_{22}} I_1 \right]$$

$$V_1 = \frac{z_{12}}{z_{22}} V_2 + \frac{z_{11} z_{22} - z_{12} z_{21}}{z_{22}} I_1 \rightarrow (6)$$

comparing (1) & (6) equations

$$h_{11} = \frac{z_{11} z_{22} - z_{12} z_{21}}{z_{22}} \quad ; \quad h_{12} = \frac{z_{12}}{z_{22}}$$

comparing (2) & (5) equations:

$$h_{21} = \frac{-z_{21}}{z_{22}} \quad ; \quad h_{22} = \frac{1}{z_{22}}$$

* h-parameters in terms of y- parameters:

h-parameter equations are:

$$V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (2).$$

y- parameter Equations:

$$I_1 = y_{11} V_1 + y_{12} V_2 \rightarrow (3)$$

$$I_2 = y_{21} V_1 + y_{22} V_2 \rightarrow (4)$$

$$(3) \Rightarrow y_{11} V_1 = I_1 - y_{12} V_2$$

$$V_1 = \frac{1}{y_{11}} I_1 - \frac{y_{12}}{y_{11}} V_2 \rightarrow (5)$$

$$(4) \Rightarrow I_2 = y_{21}\left[\frac{1}{y_{11}} I_1 - \frac{y_{12}}{y_{11}} V_2\right] + y_{22} V_2$$

$$I_2 = \frac{y_{21}}{y_{11}} I_1 + \frac{y_{11} y_{22} - y_{12} y_{21}}{y_{11}} V_2 \rightarrow (6)$$

Compare (1) & (5) equations

$$h_{11} = \frac{1}{y_{11}}, \quad h_{12} = \frac{-y_{12}}{y_{11}}$$

Compare (2) & (6) equations

$$h_{21} = \frac{y_{21}}{y_{11}}, \quad h_{22} = \frac{y_{11} y_{22} - y_{12} y_{21}}{y_{11}}$$

h parameters in terms of ABCD parameters

h parameters equations are:

$$V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (2)$$

ABCD- parameters equations are:

$$V_1 = A V_2 - B I_2 \rightarrow (3)$$

$$I_1 = C V_2 - D I_2 \rightarrow (4)$$

$$(4) \Rightarrow \quad D I_2 = C V_2 - I_1$$

$$I_2 = \frac{C}{D} V_2 - \frac{1}{D} I_1 \rightarrow (5)$$

$$(3) \Rightarrow \quad V_1 = A V_2 - B \left[ \frac{C}{D} V_2 - \frac{1}{D} I_1 \right]$$

$$V_1 = \frac{AD - BC}{D} V_2 + B/D \, I_1 \rightarrow (6).$$

compare (1) & (6) equations

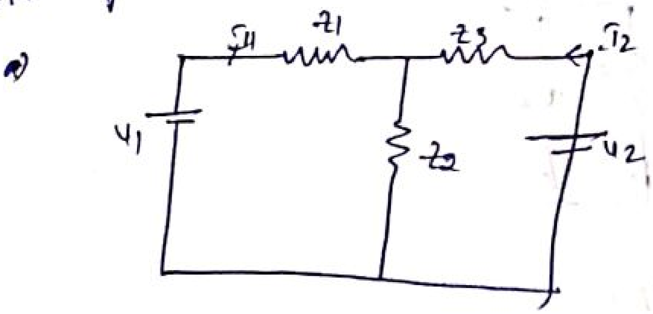
$$h_{11} = B/D \; ; \quad h_{12} = \frac{AD - BC}{D}$$

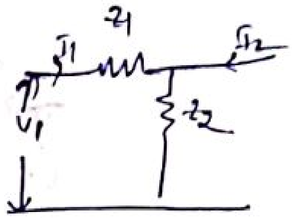
compare (2) & (5) equations

$$h_{21} = \frac{-1}{D} \; ; \quad h_{22} = C/D$$

find z-parameters

a)



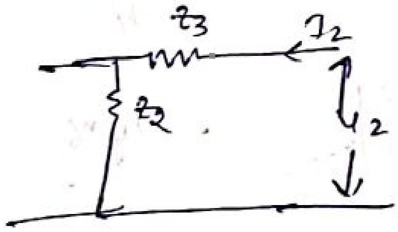




$$z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = \frac{I_1 \times z_{eq}}{I_1}$$

$= z_{eq} = z_1 + z_2$

$$z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} = \frac{I_1 \times z_2}{I_1} = z_2$$

when $I_1 = 0$




$$z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

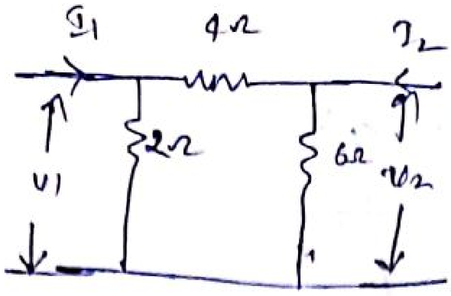
$$= \frac{I_2 \times z_{eq}}{I_2} = z_{eq}$$

$= z_2 + z_3$

$$z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

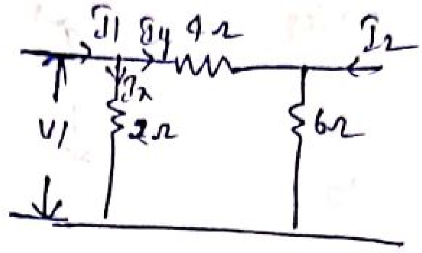
$$= \frac{I_2 \times z_{22}}{I_2} = z_{22}.$$

*




Sol:-

$I_2 = 0$ port-2 ⇒ open




$R_{eq} = (6+4) \parallel 2$

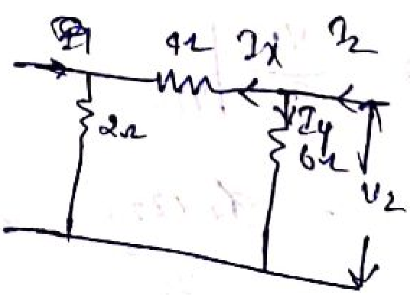
$= 10 \parallel 2 = 1.66\ \Omega$

$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{I_1 \times Z_{eq}}{I_1} = 1.66\Omega$

$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{I_y \times 6}{I_1}$

$= \frac{I_1 \times \frac{2}{2+4+6} \times 6}{I_1} = 1\Omega$

$Z_{22}$ = $I_1 = 0$ port-1 → open




$Z_{eq} = (2+4) \parallel 6$

$= 6 \parallel 6 = 3\Omega$

$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{I_2 \times Z_{eq}}{I_2}$

$= 3\Omega$

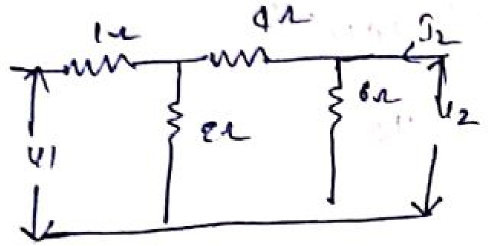
V

$$z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

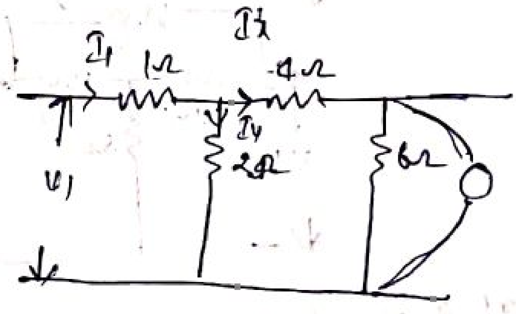
$$= \frac{I_x \cdot 2}{I_2} = \frac{I_2 \times \frac{6}{2+4+6} \times 2}{I_2}$$

$$= 1\Omega$$

*




$I_2 = 0$ port-2 ⇒ open




$$R_{eq} = ((6+4)\|2) + 1$$

$$= (10\|2) + 1$$

$$= 1.66 + 1$$

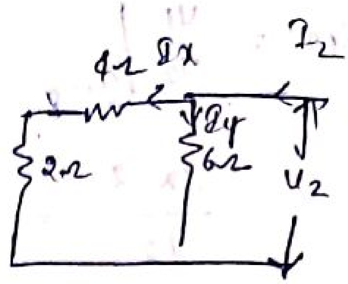
$$= 2.66\Omega$$

$$z_{11} = \frac{V_1}{I_1} = \frac{I_1 \times R_{eq}}{I_1} = 2.66\Omega$$

$$z_{21} = \frac{V_2}{I_1} = \frac{I_x \cdot 6}{I_1} = \frac{I_1 \cdot \frac{2}{2+4+6} \cdot 6}{I_1}$$

$$= 1\Omega$$

$I_1 = 0 \Rightarrow$ port (1) $\Rightarrow$ open




$R_{eq} = (2+4) \| 6$

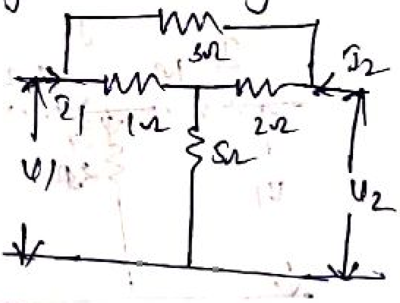
$= 3\Omega$

$z_{22} = \frac{V_2}{I_2} = \frac{I_2 \times R_{eq}}{I_2} = 3\Omega$

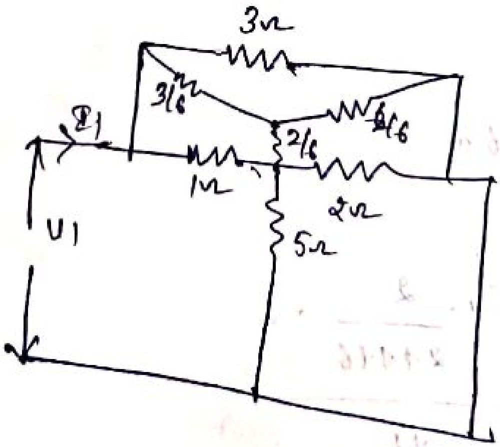
$z_{12} = \frac{V_1}{I_2} = \frac{I_x \cdot 2}{I_2} = \frac{I_2 \cdot \frac{6}{2+4+6} \cdot 2}{I_2}$

$= 1\Omega$

* obtain ~~open~~ short-circuit parameters

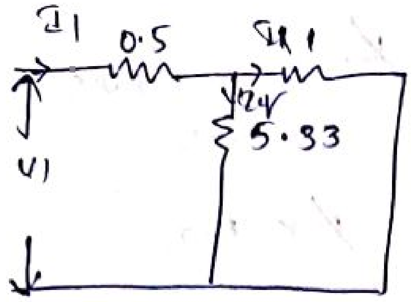



port-2 is short.




$y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = \frac{V_1/R_{eq}}{V_1}\Big|_{V_2=0}$

$y_{11} = \frac{1}{R_{eq}}$




$R_{eq} = 1.342\ \Omega$

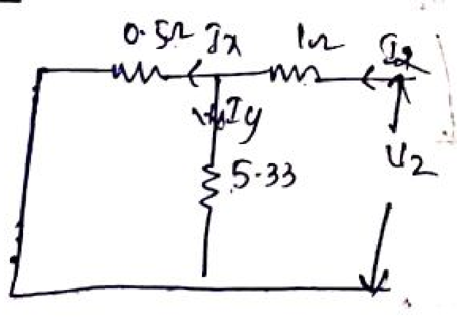
$y_{11} = \frac{1}{1.342} = 0.745\ \mho$

$y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} = \frac{I_x \cdot 1}{V_1}$

$= - \frac{\frac{I_1 \times 5.33}{5.33+1} \cdot 1}{I_1 \times R_L}$

$= \frac{5.33}{6.33 \times 1.34} = -0.62\ \mho$

port-1 is short:




$y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} = \frac{V_2 / R_{eq}}{V_2}\Big|_{V_1=0}$

$y_{22} = \frac{1}{R_{eq}} = \frac{1}{(0.5 \| 5.33)+1}$

$= \frac{1}{\ } = 0.689\ \mho$

$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

$$= \frac{I_x \cdot 0.5}{V_2} = \frac{\dfrac{I_2 \cdot 5.33}{5.33+0.5} \times 0.5}{I_2 \times R_{eq}}$$

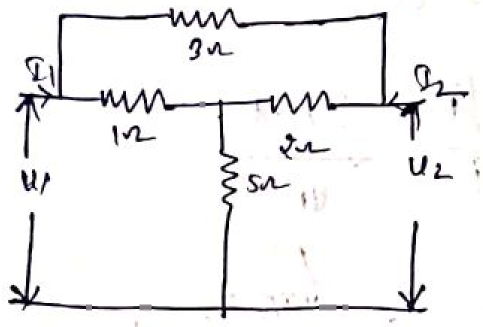
$$= \frac{0.457}{1.45}$$

$$= -0.330\,\mho$$

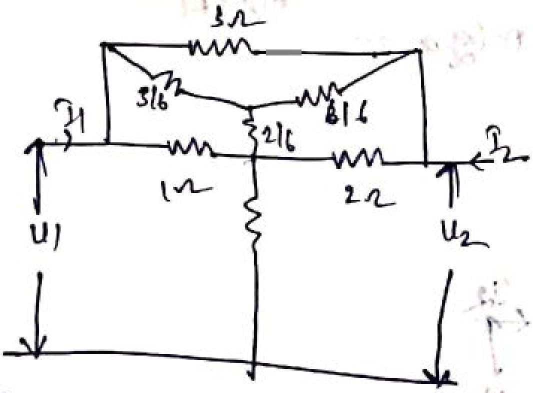
$$\begin{bmatrix} \frac{35}{6}, & \frac{16}{3} \\ \frac{16}{3}, & \frac{19}{3} \end{bmatrix}$$

$0 : 8.5$

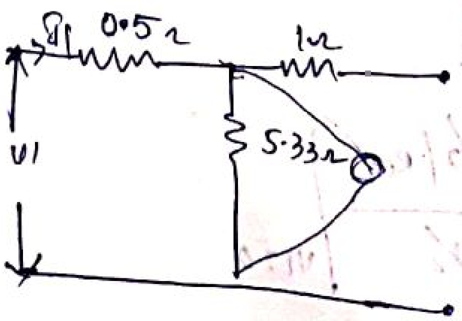
*




find z-parameters




port-2 is open $I_2 = 0$



$$R_{eq} = 0.5 + 5.33$$
$$= 5.83$$

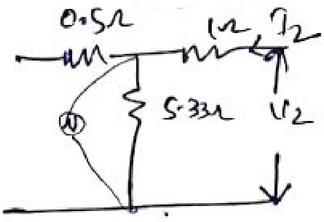
$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{I_1 \times R_{eq}}{I_1} = 5.83\,\Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{I_2 \cdot 5.33}{I_1}$$

$$= \frac{I_1 \cdot 5.33}{I_1} = 5.33\,\Omega$$

port-1 is open $I_1 = 0$

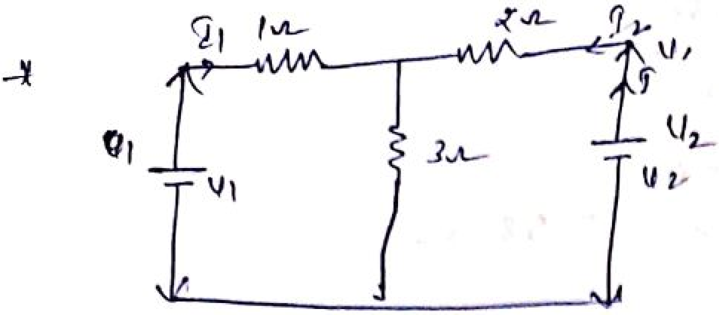
$Z_{22} = \frac{V_2}{}$




$$Z_{22} = \frac{V_2}{I_2} = \quad R_{eq} = 1 + 5.33$$
$$= 6.33$$

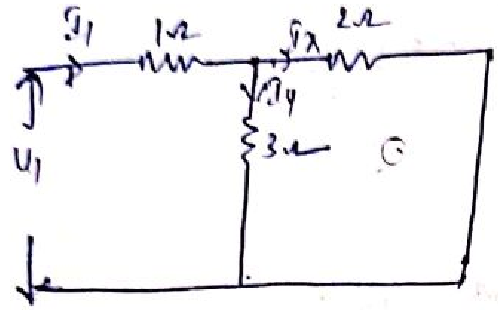
$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{I_2 \cdot R_{eq}}{I_2} = 6.33\,\Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{I_2 \cdot 5.33}{I_2} = 5.33\,\Omega$$




. find Y-parameters.

sol. $V_2 = 0$ ; port 2 is short circuited




$$R_{eq} = (2\|3)+1$$

$$= \frac{2\times3}{2+3}+1 = \frac{6}{5}+1$$

$$= 11/5$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{V_1/R_{eq}}{V_1} = \frac{1}{R_{eq}}$$

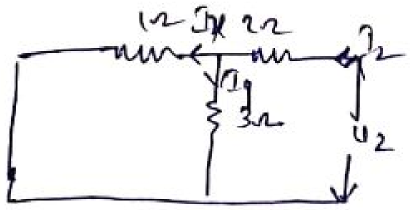
$$= 11/5 \ \mho$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{I_x \cdot 2}{V_1}$$

$$= \frac{I_1 \cdot \frac{3}{3+2} \cdot 2}{I_1 / R_{eq}}$$

$$= \frac{3}{5} \cdot R_{eq}$$

$$= \frac{3}{5} \times \frac{5}{11} = \frac{3}{11} \mho$$




port-1 is short circuited

$V_1 = 0$

$R_{eq} = (1 \| 3) + 2 = \frac{3}{4} + 2 = 11/4\ \Omega$

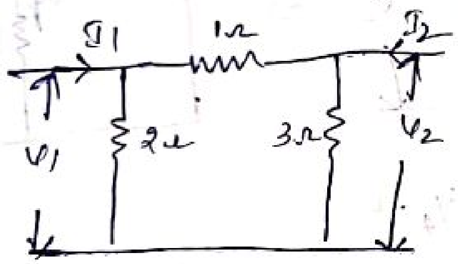
$Y_{22} = \frac{1}{R_{eq}} = 4/11\ \mho$

$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{I_x \cdot}{V_2} = \frac{I_2 \cdot \frac{3}{1+3}}{I_2 \cdot R_{eq}}$

$= \frac{3}{4} \cdot \frac{4}{11} = -3/11\ \mho$

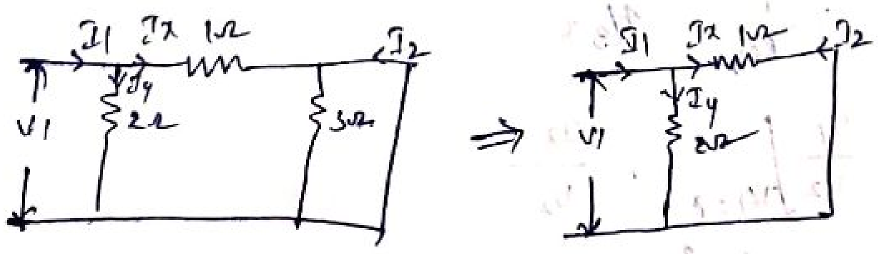
[-ve due to opposite direction of current]

\+




find Y-parameters.

Sol. when $V_2 = 0$; port-2 is short ckt




$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} =$

$R_{eq} = 1 \parallel 2$

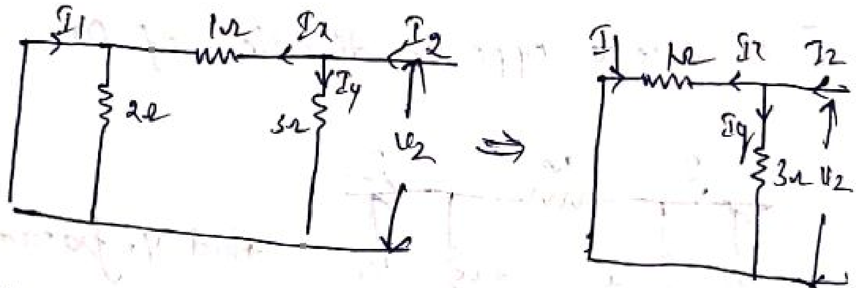
$= \frac{1 \times 2}{1+2} = \frac{2}{3}$

$Y_{11} = \frac{1}{R_{eq}} = \frac{1}{2/3} = 3/2 \, ℧$

$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} = \frac{-I_x}{V_1}$

$= \frac{I_1 \cdot \frac{2}{1+2}}{I_1 \cdot R_{eq}} = \frac{2/3}{2/3} = -1 \, ℧$

when $V_1 = 0$ Port -1 is short circuited



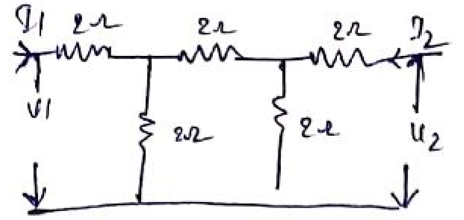

$R_{eq} = 1 \parallel 3 = \frac{1 \times 3}{1+3} = 3/4 \, Ω$

$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} = \frac{V_2 / R_{eq}}{V_2}$

$= \frac{1}{R_{eq}} = 4/3 \, ℧$

$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} = \frac{-I_x}{V_2}$

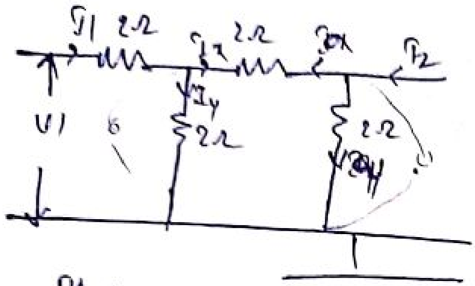
$= \frac{I_2 \cdot \frac{3}{1+3}}{I_2 \cdot R_{eq}} = \frac{3/4}{3/4} = -1 \, ℧$




find Z & Y parameters.

## Z parameters

If $I_2 = 0$; port 2 is open circuited




$$z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = \frac{V_1/R_{eq}}{V_1} = \frac{1}{\frac{1}{R_{eq}}} = R_{eq}$$

$$R_{eq} = ((2+2)\,\|\,2)+2$$

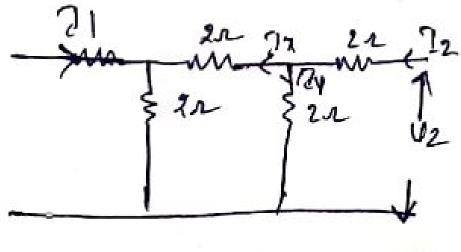
$$= 10/3$$

$$z_{11} = 10/3\ \Omega$$

$$z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} = \frac{-I_x \cdot 2}{I_1}$$

$$= \frac{-I_1 \cdot \frac{2}{2+2+2} \cdot 2}{I_1} = \frac{-4}{6} = -2/3\ \Omega$$

If $I_1 = 0$; port 1 is open circuit



$$R_{eq} = ((2+2)\|2)+2$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = R_{eq}$$

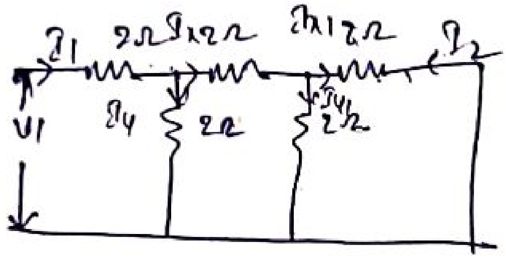
$$= 10/3\ \Omega$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{-I_x \cdot 2}{I_2}$$

$$= \frac{-I_2 \cdot \frac{2}{2+2+2} \cdot 2}{I_2}$$

$$= -4/6 = -2/3\ \Omega$$

## Y- Parameters




$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$$Y_{11} = \frac{1}{R_{eq}}$$

$$R_{eq} = (((2\|2)+2)\|2)+2$$

$$= \frac{3 \times 2}{3+2} + 2 = \frac{6}{5} + 2$$

$$= 16/5$$

$$Y_{11} = \frac{5}{16}$$

$$I_x = I_1 \cdot \frac{2}{2+2+2}$$

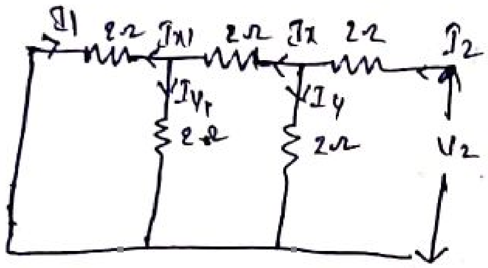
$$= \frac{2I_1}{6} = \frac{I_1}{3}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{I_{x1}}{V_1}$$

$$= \frac{-\frac{I_1}{3} \cdot \frac{2}{2+2}}{I_1 \cdot R_{eq}} = \frac{-\frac{2}{12}}{16/5}$$

$$= -\frac{2}{12} \times \frac{5}{16} = \frac{-5}{16 \times 6}\ \mho$$

$V_1 = 0$




$$R_{eq} = \left[ \left( (2 \| 2) + 2 \right) \| 2 \right] + 2$$

$$= 16/5\ \Omega$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{V_2 / R_{eq}}{V_2}$$

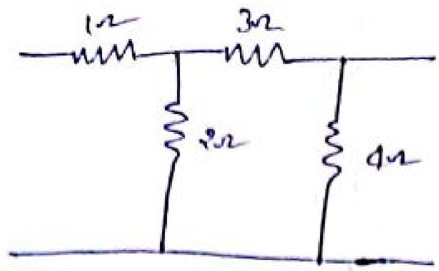
$$= \frac{1}{R_{eq}} = 5/16$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \left.\frac{-I_{x1}}{V_2}\right|_{V_1=0}$$

$$I_x = \frac{I_2 \cdot 2}{2+2+2} = \frac{I_2}{3}$$

$$Y_{12} = \frac{-\frac{I_2}{3} \cdot \frac{2}{2+2}}{I_2 R_{eq}} = \frac{-\frac{2}{12}}{16/5} = \frac{-5}{16 \times 6}\ \mho$$

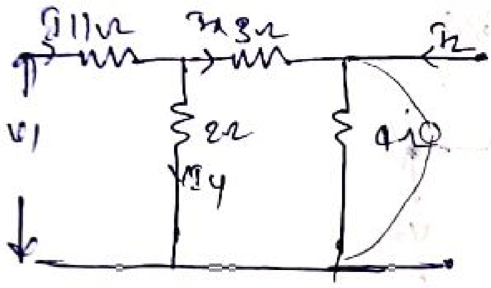
* Find ABCD-parameters for the given Network:




$$V_1 = A V_2 - B I_2$$

$$I_1 = C V_2 - D I_2$$

If $I_2 = 0$, the port-2 is open ckt




$$A = V_1/V_2 \Big|_{I_2=0} = \frac{I_1 \cdot R_{eq}}{I_x \cdot 4\Omega}$$

$$R_{eq} = ((3+4) \parallel 2) + 1$$

$$= \frac{7 \times 2}{7+2} + 1 = \frac{14}{9} + 1$$

$$= \frac{23}{9}\,\Omega$$

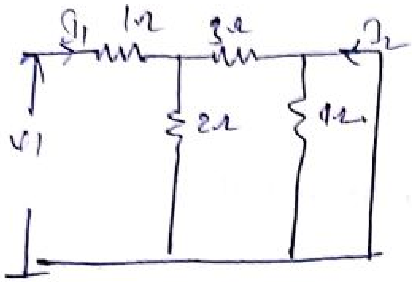
$$I_x = \frac{I_1 \cdot 2}{2+3+4} = \frac{2I_1}{9}$$

$$A = \frac{I_1 \cdot 23/9}{\frac{2I_1}{9} \cdot 4} = \frac{23}{8}\,\mho$$

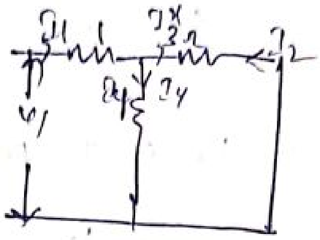
$$C = \frac{I_1}{V_2}\Big|_{I_2=0} = \frac{I_1}{I_x \cdot 4}$$

$$= \frac{I_1}{\frac{I_1 \cdot 2}{2+3+4} \cdot 4} = \frac{9}{8}\ \mho$$

If $V_2 = 0$ the port-2 is short circuit




$\Rightarrow$




$$B = \frac{-V_1}{I_2}\Big|_{V_2=0}$$

$$= \frac{-I_1 \cdot R_{eq}}{I_2} = \frac{+I_1 \cdot R_{eq}}{+I_y}$$

$$= \frac{I_1 \cdot R_{eq}}{I_y}$$

$$R_{eq} = (3 \| 2) + 1 = \frac{3 \times 2}{3+2} + 1$$

$$= \frac{6}{5} + 1 = \frac{11}{5}$$

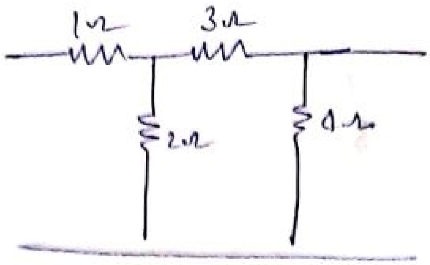
$$I_y = \frac{I_1 \cdot 2}{3+2} = \frac{2I_1}{5}$$

$$B = \frac{I_1 \cdot 11/5}{2I_1/5} = 11/2\ \Omega$$

$$D = -I_1/I_2 = \frac{+I_1}{+I_y} = I_1 \Big/ \frac{2I_1}{5}$$

$$= 5/2\ \Omega$$

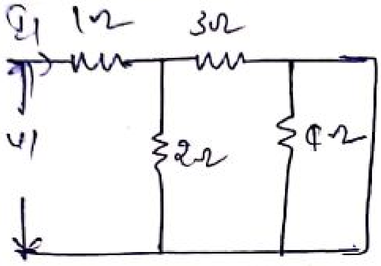
H-parameters for this circuit



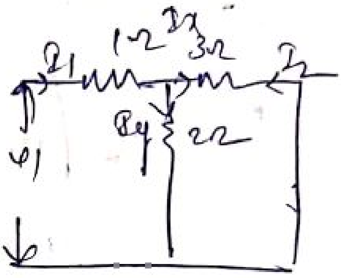

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

when $V_2 = 0$ the port -2 is shortcircuited




⇒




$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0} = \frac{I_1 \cdot R_{eq}}{I_1}$

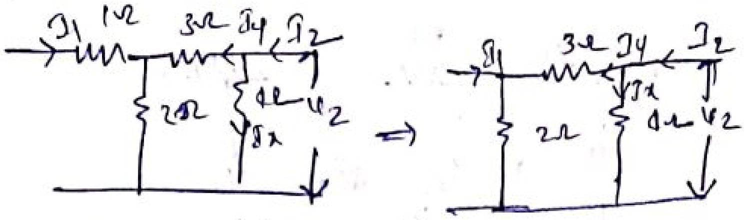
$R_{eq} = (3\|2) + 1 = \frac{6}{5} + 1 = 11/5$

$h_{11} = R_{eq} = 11/5\ \Omega$

$h_{21} = \frac{I_2}{I_1} = \frac{-I_x}{I_1}$

$= \frac{-I_1 \cdot \frac{2}{2+3}}{I_1} = -2/5$

when $I_1 = 0$: port 1 is open ckt




$$h_{12} = \frac{V_1}{V_2} = \frac{I_x}{-I_y} \quad \frac{I_y \cdot 2}{I_2 \cdot R_{eq}}$$

$$R_{eq} = (2+3) \| 4$$

$$= 5 \| 4 = \frac{5 \times 4}{5+4} = \frac{20}{9}\ \Omega$$

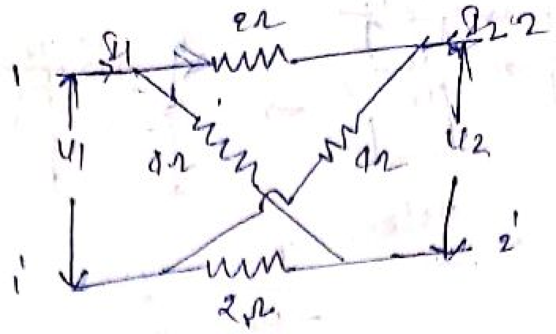
$$I_y = \frac{I_2 \cdot 4}{2+3+4} = \frac{4I_2}{9}$$

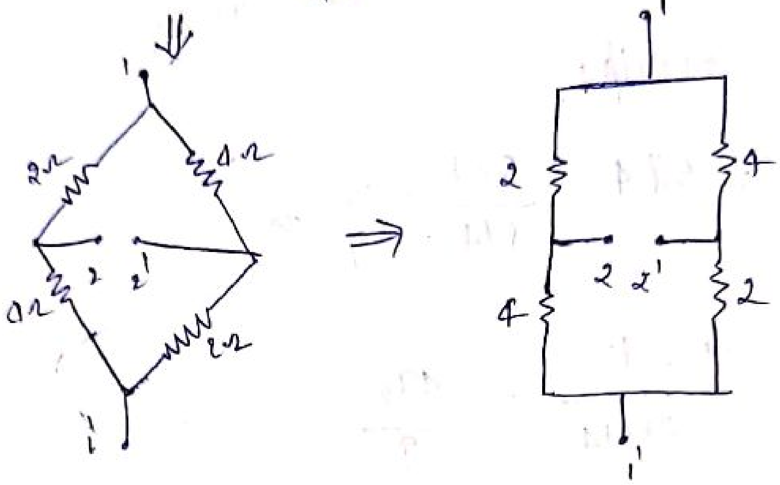
$$h_{12} = \frac{\frac{4I_2}{9} \cdot 2}{I_2 \cdot \frac{20}{9}} = \frac{2}{5}$$

$$h_{22} = \frac{I_2}{V_2} = \frac{I_2}{I_2 \cdot R_{eq}}$$

$$= \frac{1}{R_{eq}} = \frac{1}{20/9} = \frac{9}{20}\ \mho$$

* find z, y, ABCD, h- parameters for given circuit.



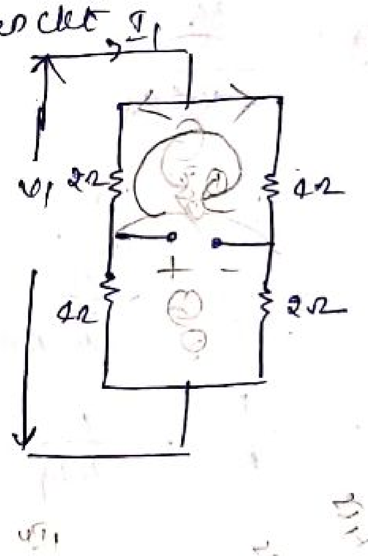


z parameters :-

when $I_2 = 0$ port-2 open ckt

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

$$= \frac{I_1 \cdot R_{eq}}{I_1} = R_{eq}$$



$$R_{eq} = (2+4) \| (2+4)$$

$$= 6 \| 6 = 3\Omega$$

$$Z_{11} = 3\Omega$$

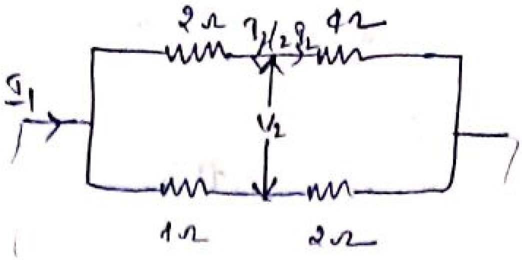
$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$$

$V_2$ = Vdrop at $2\Omega$ + Vdrop at $4\Omega$

$$= 2 \cdot \frac{I_1}{2} + 4\frac{I_1}{2} \quad = 3I_1$$

$$Z_{21} = 1\Omega$$

when $I_1=0$ port-1 is open circuit.

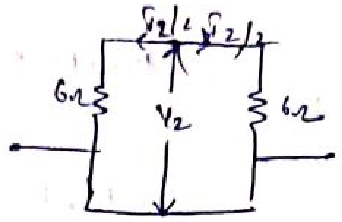


$$z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} = \frac{I_2 \cdot R_{eq}}{I_2}$$

$= R_{eq}$

$R_{eq} = (2+4)\,||\,(2+4)$

$= 3\Omega$



$z_{22} = 3\Omega$

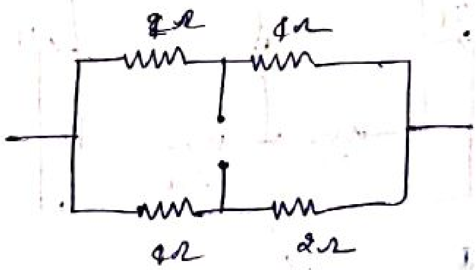
$$z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} = \frac{6\,I_2/2}{I_2}$$

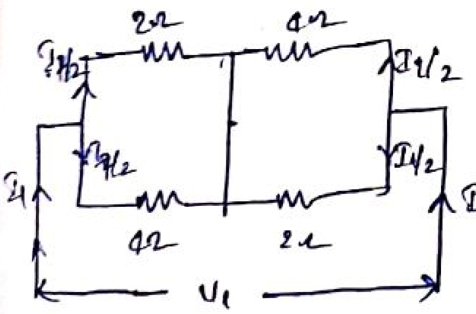
$= 3\Omega$

$\frac{6 I_2}{2} - \frac{2 I_2}{2} = I_2$

## Y-parameters:-

when $V_2 = 0$, the port-2 is short circuited





$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = \frac{V_1/R_{eq}}{V_1}$$

$= 1/R_{eq}$ ℧

$R_{eq} = (2||4)\,||\,(2||4)$

$= \frac{2\times4}{2+4}\,||\,\frac{2\times4}{2+4} = \frac{4}{3}\,||\,\frac{4}{3}$

$$= \frac{4/3 \times 4/3}{\frac{4}{3}+\frac{4}{3}} = \frac{\frac{16}{9}}{\frac{2}{3}}$$

$$= \frac{16}{9} \times \frac{3}{2} = 8/3\ \Omega$$

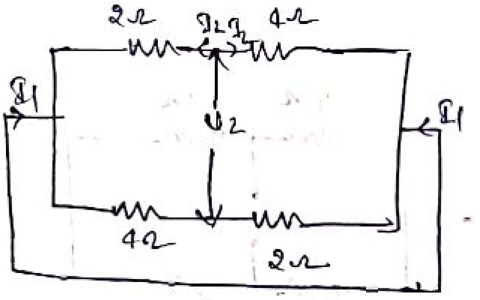
$$Y_{11} = \frac{1}{R_{eq}} = \frac{1}{8/3} = \frac{3}{8}\ ℧$$

$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} = \frac{\text{current through } 2\Omega + \text{current through } 4\Omega}{V_1}$$

$$= \frac{I_1/2 + I_1/2}{I_1 \cdot R_{eq}} = \frac{I_1}{R_{eq} \cdot I_1}$$

$$= \frac{1}{R_{eq}} = \frac{3}{8}\ ℧ \quad = -1/3$$

when $V_1 = 0$, the port-1 is short circuited.




$$Y_{22} = \frac{I_2}{V_2} = \frac{\frac{1}{I/2 \cdot R_{eq}}}{I/2} = \frac{1}{R_{eq}}$$

$$R_{eq} = (2 \| 4) \| (2 \| 4)$$

$$= \left(\frac{2\times 4}{2+4}\right) \| \left(\frac{2\times 4}{2+4}\right)$$

$$= \frac{4}{3} \| 4/3 = \frac{\frac{4}{3}\times\frac{4}{3}}{\frac{4}{3}+\frac{4}{3}} = \frac{\frac{16}{9}}{2/3}$$

$$= \frac{16}{9} \times \frac{3}{2} = 8/3$$

$Y_{22} = 1/8/3 = 3/8$ ℧

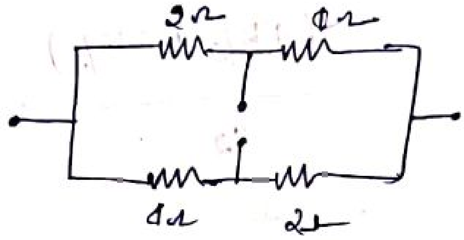
$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$

$= \frac{\text{current through } 2\Omega + \text{current through } 4\Omega}{V_2}$

$= \frac{I_2/2 + I_2/2}{I_2 \cdot R_{eq}} = \frac{I_2}{I_2 \cdot R_{eq}}$

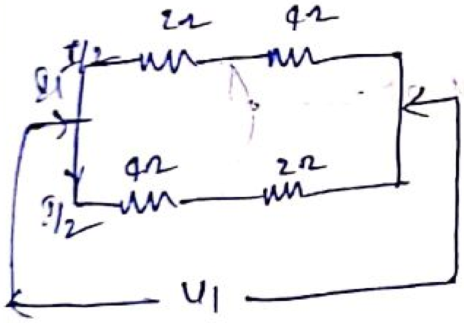
$= 1/R_{eq} = 3/8$ ℧ $= -1/3$

ABCD-parameters :-




when $I_2 = 0$

port-2 open circuited




$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

$A = \frac{V_1}{V_2} = \frac{I_1 \cdot R_{eq}}{\text{V drop at } 4\Omega + \text{V drop at } 2\Omega}$

$R_{eq} = (2+4) \| (2+4)$

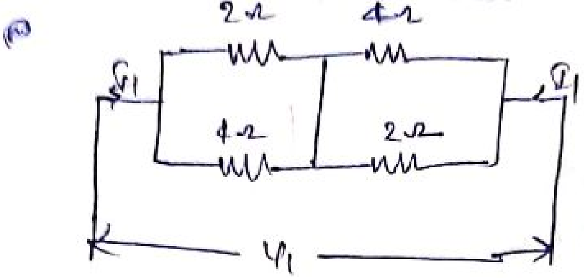
$= 6 \| 6 = 3\Omega$

$V_2 = 4I_1/2 - 4I_1/2 = \frac{2I_1 - I_1}{} = I_1$

$A = \frac{I_1 \cdot 3\Omega}{I_1} = 3$

$C = \frac{I_1}{V_2} = \frac{I_1}{8I_1} = \frac{1}{8}$ ℧

When $V_2 = 0$ the port 2 is short circuited




$B = \frac{-V_1}{I_2} = \frac{-(V \text{ drop at } 4\Omega + V \text{ drop at } 2\Omega)}{I_2}$

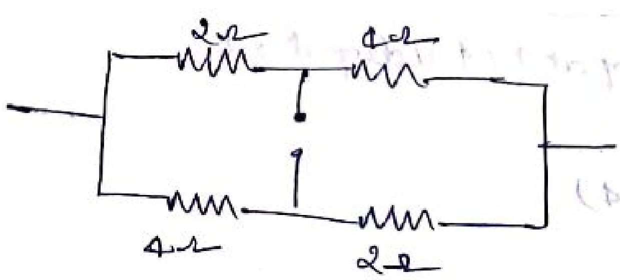
$= \frac{-(4I_2/2 + 2I_2/2)}{I_2} = \frac{-3I_2}{I_2}$

$= -3\ \Omega$

$D = \frac{-I_1}{I_2} = \frac{-I_1}{\text{current at } 4\Omega + \text{current at } 2\Omega}$

$= \frac{-I_1}{\frac{I_1}{2} + \frac{I_1}{2}} = -1$

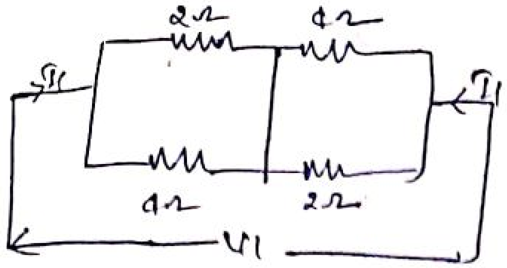
H - parameters:-




$V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

when $V_2=0$ port 2 is short circuited




$h_{11} = \frac{V_1}{I_1}$

$= \frac{I_1 \cdot R_{eq}}{I_1} = R_{eq}$

$R_{eq} = (2||4)||(2||4)$

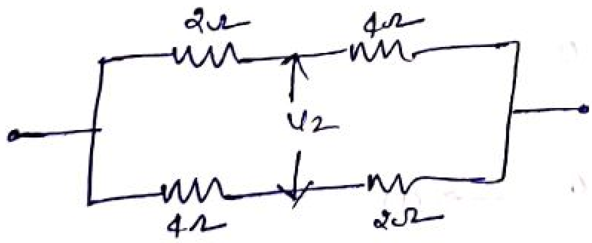
$= 8/3$

$h_{11} = 8/3\ \Omega$

$h_{21} = \frac{I_2}{I_1} = \frac{\text{current through } 2\Omega + \text{current through } 4\Omega}{I_1}$

$= \frac{I_1/2 + I_1/2}{I_1} = \frac{I_1}{I_1} = 1 \cdot \quad 1/3$

when $I_1=0$ port (1) is open circuit




$h_{22} = \frac{I_2}{V_2} = \frac{I_2}{I_2 \cdot R_{eq}} = 1/R_{eq}$

$R_{eq} = (2+4)||(2+4) = 6||6$

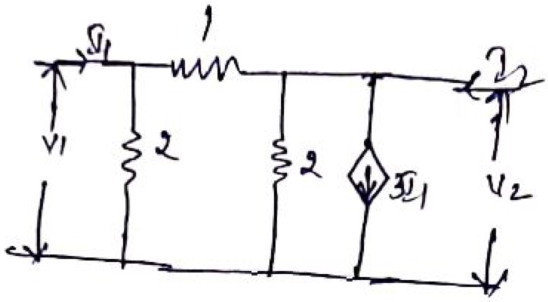
$= 3\Omega$

 $h_{22} = \frac{1}{3}\ \mho$

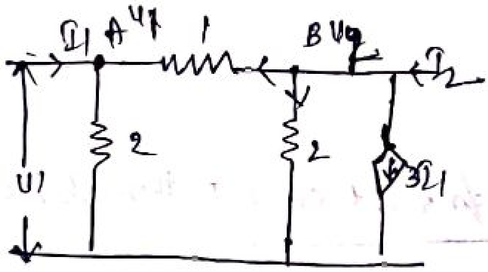
$$h_{21} = \frac{I_2}{I_1} = \frac{I_2}{I_2/2 + I_2/2} = \frac{I_2}{I_2}$$

$= 1$

* find z & y - parameters




when $I_2 = 0$, port-2 open




Current from high potential to low potential

apply KCL at node B

$$I_2 = 3I_1 + \frac{V_2}{2} + \frac{V_2 - V_1}{1}$$

$$3I_1 + \frac{V_2}{2} + V_2 - V_1 = 0$$

$$3I_1 + \frac{3V_2}{2} - V_1 = 0 \rightarrow (1)$$

apply KCL at node A

$$I_1 = \frac{V_1}{2} + \frac{V_1 - V_2}{1} \rightarrow (2)$$

Substitute (2) in (1).

$$3\left[\frac{3V_1}{2} - V_2\right] + \frac{3V_2}{2} - V_1 = 0$$

$$\frac{9V_1}{2} - 3V_2 + \frac{3V_2}{2} - V_1 = 0$$

$$\frac{7V_1}{2} - \frac{5V_2}{2} = 0$$

$$3I_1 + \frac{3}{2}\left[\frac{3V_1}{2} - I_1\right] - V_1 = 0$$

$$3I_1 + \frac{9}{4}V_1 - \frac{3}{2}I_1 - V_1 = 0$$

$$\frac{3}{2}I_1 + \frac{5}{4}V_1 = 0$$

$$\frac{3}{2}I_1 = -\frac{5}{4}V_1$$

$$\frac{3}{2} \cdot \frac{4}{5} = \frac{V_1}{I_1}$$

$$\frac{-6}{5} = V_1/I_1 = -6/5$$

$$\boxed{Z_{11} = -6/5}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

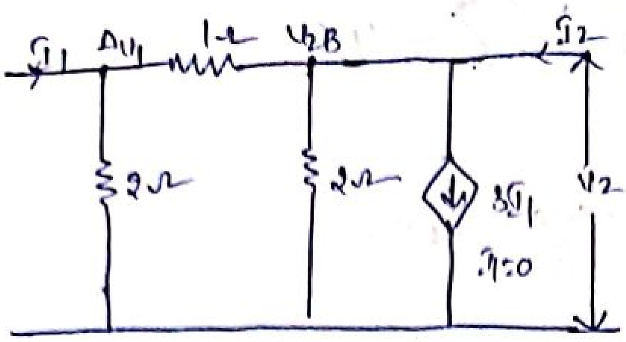
$$I_1 = \left[\frac{3I_1 + \frac{3V_2}{2}}{2}\right] + \left[3I_1 + \frac{3V_2}{2}\right] - V_2$$

$$= \frac{3I_1}{2} + \frac{3V_2}{4} + 3I_1 + \frac{3V_2}{2} - V_2$$

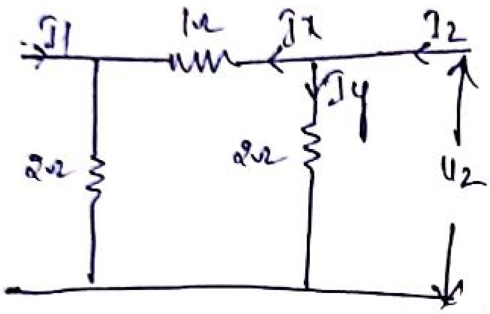
$$I_1 = \frac{9I_1}{2} + \frac{5V_2}{4}$$

$$\frac{-7I_1}{2} = \frac{5V_2}{4} \Rightarrow \frac{V_2}{I_1} = \frac{-7 \times 4^2}{2 \times 5} = -14/5$$

when $I_1 = 0$; port-1 is open ckt




⇓




$$z_{22} = \frac{V_2}{I_2} = \frac{R_{eq} \cdot I_2}{I_2}$$

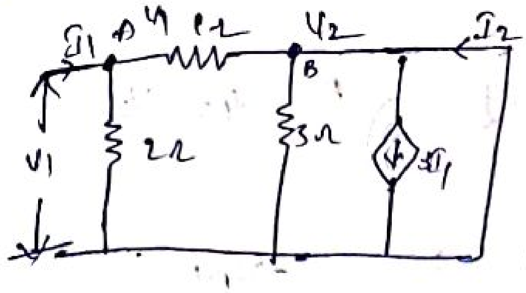
$$R_{eq} = (1+2) \| 2 = \frac{3 \times 2}{3+2} = 6/5$$

$$z_{22} = 6/5\ \Omega$$

$$z_{12} = \frac{V_1}{I_2} = \frac{I_x \cdot 2}{I_2} = \frac{I_2 \cdot \frac{2}{1+2+2} \cdot 2}{I_2}$$

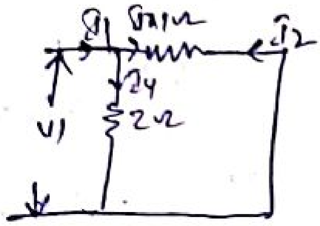
$$z_{12} = \frac{4}{5}\ \Omega$$

Y-parameters      when $V_2 = 0$; port-2 is short ckt




⇓

apply



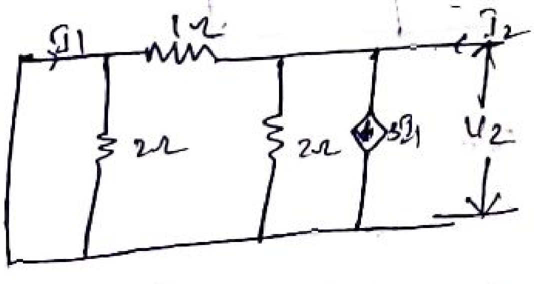

$Y_{11} = \frac{I_1}{V_1} = \frac{V_1/R_{eq}}{V_1} = \frac{1}{R_{eq}}$

$R_{eq} = 1 \| 2 = \frac{2}{3}$

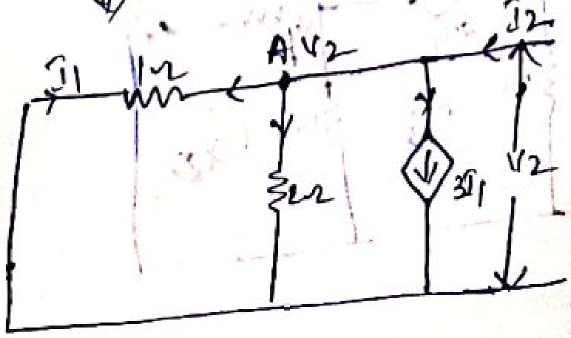
$Y_{11} = 3/2$ ℧

$Y_{21} = \frac{I_2}{V_1} = \frac{-I_x}{I_1/R_{eq}} = \frac{-I_1 \cdot \frac{2}{1+2}}{I_1 / \frac{2}{3}} = -1$ ℧

when $V_1 = 0$; port-1 is short circuit




⇓



Apply KCL at node A:

$$\frac{V_2}{1} + \frac{V_2}{2} + 3I_1 = I_2 \qquad \text{But } I_1 = -\frac{V_2}{1}$$

$$\frac{V_2}{1} + \frac{V_2}{2} - 3V_2 = I_2$$

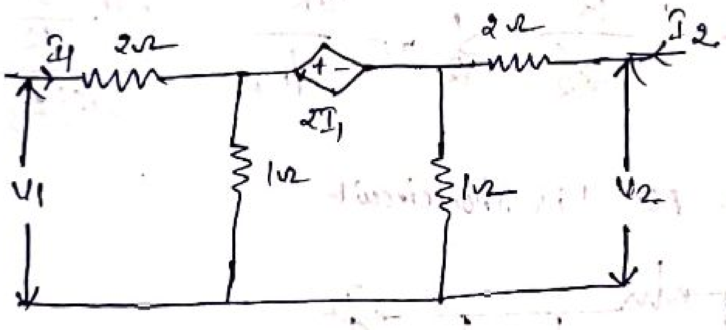
$$\frac{3V_2}{2} - 3V_2 = I_2$$

$$-\frac{3V_2}{2} = I_2$$

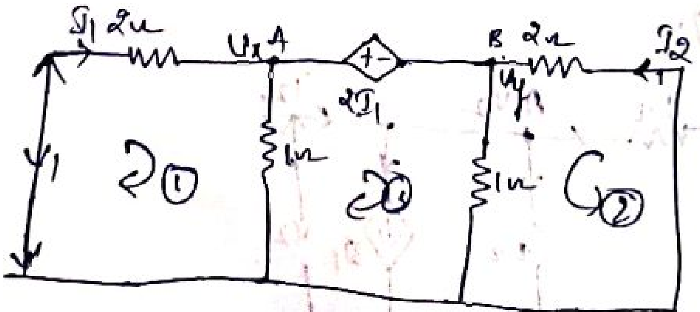
$$Y_{12} = \frac{I_1}{V_2} = \frac{-V_2}{V_2} = -1\,℧$$

$$Y_{22} = \frac{I_2}{V_2} = \frac{I_2}{V_2} = -3/2\,℧$$

* Find Y-parameters then find H-parameters.




when $V_2 = 0$; port 2 is short ckt



$I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$

Apply KVL for the loops

loop 1 - $3I_1 - I_3 = V_1 \rightarrow (1)$

loop 2 - $3I_2 + I_3 = 0 \rightarrow (2)$

$I_3 = -3I_2$

loop 3 - $2I_3 + I_2 - I_1 = -2I_1 \rightarrow (3)$

(1) $\Rightarrow$ $3I_1 - (-3I_2) = V_1$

$\boxed{3I_1 + 3I_2 = V_1} \rightarrow (4)$

(3) $\Rightarrow$ $2(-3I_2) + I_2 - I_1 + 2I_1 = 0$

$-6I_2 + I_2 + I_1 = 0$

$\boxed{I_1 - 5I_2 = 0} \rightarrow (5)$

using (4) & (5) eqn's

$Y_{11} = \frac{I_1}{V_1}$

(5) $\Rightarrow$ $5I_2 = I_1$

$I_2 = I_1/5$

$3I_1 + 3[I_1/5] = V_1$

$\frac{18I_1}{5} = V_1$

$Y_{11} = \frac{I_1}{V_1} = \frac{5}{18}\,\mho$

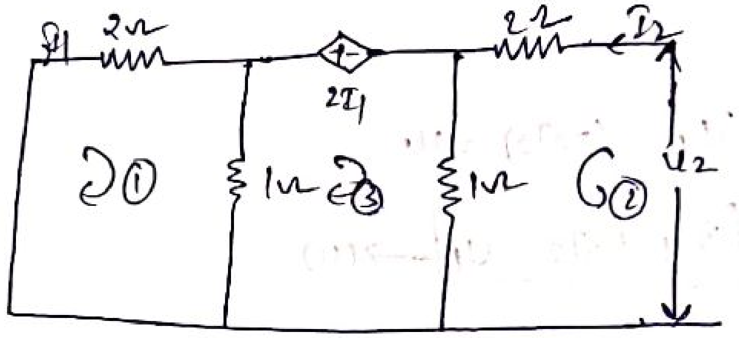
$$Y_{21} = \frac{I_2}{V_1} \quad \text{using (4) \& (5) equations}$$

$$\therefore 15I_2 + 3I_2 = V_1$$

$$18I_2 = V_1$$

$$Y_{21} = \frac{I_2}{V_1} = \frac{1}{18}\ \mho$$

when $V_1 = 0$; port-(1) is short ckt




Apply KVL for loops

Loop-1 $\Rightarrow$ $3I_1 - I_3 = 0$

$$I_3 = 3I_1 \longrightarrow (1)$$

Loop-2 $\Rightarrow$ $3I_2 - I_3 = V_2$

$$3I_2 - 3I_1 = V_2 \longrightarrow (2)$$

Loop-3 $\Rightarrow$ $2I_3 + I_2 - I_1 = -2I_1$

$$2(3I_1) + I_2 - I_1 = -2I_1$$

$$5I_1 + I_2 = 0 \longrightarrow (3)$$

$Y_{22} = \frac{I_2}{V_2}$

from eqn (2) & (3)

$$3I_2 - 3\left[\frac{-I_2}{5}\right] = V_2$$

$$3I_2 + \frac{3I_2}{5} = V_2$$

$$\frac{18I_2}{5} = V_2$$

$$Y_{22} = \frac{I_2}{V_2} = \frac{5}{18}\ \mho$$

$Y_{12} =$ from eqn (2) & (3)

$$= \frac{I_1}{V_2}$$

$$3(-5I_1) - 3I_1 = V_2$$

$$-18I_1 = V_2$$

$$\boxed{Y_{12} = \frac{I_1}{V_2} = \frac{-1}{18}\ \mho}$$

$$h_{11} = \frac{1}{Y_{11}} = \frac{18}{5}\ \Omega \qquad h_{21} = \frac{Y_{21}}{Y_{11}} = \frac{1/18}{5/18} = \frac{1}{5}$$

$$h_{12} = \frac{-Y_{12}}{Y_{11}} = \frac{-(-1/18)}{5/18} = 1/5 \qquad h_{22} = \frac{Y_{11}Y_{22} - Y_{21}Y_{12}}{Y_{11}} = \frac{\frac{5}{18}\cdot\frac{5}{18} - \frac{1}{18}\cdot\frac{1}{18}}{5/18}$$

$$= \frac{4}{18}\Big/\frac{5}{18} = 4/5\ \mho$$